PY4612 ADVANCED LOGIC

Departments of Philosophy
University of St Andrews

Candlemas 2013-2014

Module Coordinator: Dr Aaron J. Cotnoir
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Office: Edgecliffe G07
Office Hours: Mondays, 10.00–11.00 (or by appointment)

Lectures: every Monday, 14.00–16.00, Arts Seminar 4
Tutorials: weekly, starting week two (sign up via MMS)

Group 1: Tues, 10.00–11.00, Edgecliffe 104

Group 2: Tues, 11.00–12.00, Edgecliffe 104

Course Description

The module presupposes facility in the elementary practice of logic provided by PY2001. Previous logic modules have been focused on making use of particular types of logic. This module makes use of metatheoretical techniques to make logic itself the subject of formal investigation. The main goals of the module will be to tackle the standard metatheoretical results: completeness, compactness, the Lowenheim-Skolem theorems, and Gödel's celebrated incompleteness theorems. Along the way, there will be preparatory discussion of elementary set theory, model theory, and recursion theory.

Learning Outcomes

By the end of the module, students will have gained an understanding of some of the most central and historically influential results in mathematical logic. You will gain proficiency in set-theoretic and model-theoretic techniques, including the ubiquitous proof by mathematical induction. By studying the properties of logical systems themselves, you will develop skills useful for work in philosophical logic, philosophy of mathematics, metaphysics, and philosophy of language.

Assessment

Workload This module is worth 30 credits; hence it should typically occupy half of your working week. The standard university working week is 37.5 hours, so this module should occupy about 18 hours per week, of which only three are spent in class. This means you should plan to spend 15 hours per week devoted to reading, rereading, and doing exercises outside of class. This is an extremely challenging course in which both the methodology, techniques, and results themselves are developed at a level of abstraction you may not have previously encountered. Please plan your study habits accordingly.

Weighting and components 50% coursework, 50% exam, mandatory tutorials. You will pass the module only if you turn in all coursework, take the exam, and miss at most two tutorial sessions. Reassessment is permitted for those who fail with an grade of > 4.0.

- *N.B. three or more unexcused absences from tutorials will result in failure with a grade of '0X', which does not permit reassessment.*

Coursework All readings should be done in advance of the lecture. The primary assessed coursework for this module will be exercise sets for each chapter. These are to be turned in at the start of each tutorial, and will be worth a maximum of 2 points toward your coursework grade. Each week's exercises will be marked as *excellent* (2 points), *satisfactory* (1 point), or *unsatisfactory* (0 points). Failure to turn in exercises before the start of tutorial means you forfeit that week's points. Any unexcused tutorial absence will likewise lose you 2 points.

Exam The exam will last three hours and will consist of a number of short-answer questions on topics from throughout the semester. More details will be provided in revision week.

Textbooks We will be using (BBJ):

> G. Boolos, J. Burgess, and R. Jeffrey's *Computability and Logic* 5th edition, Cambridge University Press, 2007 [ISBN: 0521701465].[1]

Some supplementary texts:

> G. Hunter, *Metalogic*, University of California Press, 1996.
>
> S. Hedman, *A First Course in Logic*, Oxford University Press, 2004.[2]
>
> H. Enderton, *An Introduction to Mathematical Logic*, 2nd edition, Harcourt / Academic Press, 2001.

For more details please read the *Philosophy Handbook for Undergraduates* carefully regarding absences, late assignments, academic alerts, plagiarism etc. Ignorance of the information in the handbook does not excuse you from failing to meet module requirements.
See: `http://www.st-andrews.ac.uk/philosophy/current/ugrad/`

[1] The 4th edition [ISBN: 0521007585] is also acceptable; but please note that the 3rd edition is *not*.

[2] This is an excellent supplementary resource for those with a background in mathematics and/or computer science.

Provisional Schedule

WEEK 1 **Enumeration & Diagonalization**
Required Reading: BBJ chs. 1–2
Mon. Jan. 27 Lecture
Tues. Jan. 28 No Tutorial
(Exercises unassessed)

WEEK 2 **First-Order Logic**
Required Reading: BBJ chs. 9–10
Mon. Feb. 03 Lecture
Tues. Feb. 04 Tutorial

WEEK 3 **Lowenheim-Skolem Thms**
Required Reading: BBJ ch. 12
Mon. Feb. 10 Lecture
Tues. Feb. 11 Tutorial

WEEK 4 **Compactness**
Required Reading: BBJ ch. 13
Mon. Feb. 17 Lecture
Tues. Feb. 18 Tutorial

WEEK 5 **Completeness**
Required Reading: BBJ ch. 14
Mon. Feb. 24 Lecture
Tues. Feb. 25 Tutorial

WEEK 6 **Recursive Functions**
Required Reading: BBJ ch. 6
Mon. Mar. 10 Lecture
Tues. Mar. 11 Tutorial

WEEK 7 **Recursive Sets & Relations**
Required Reading: BBJ ch. 7
Mon. Mar. 03 Lecture
Tues. Mar. 04 Tutorial

BREAK Mon. Mar. 17— Fri Mar. 28

WEEK 8 **Arithmetization**
Required Reading: BBJ ch. 15
Mon. Mar. 31 Lecture
Tues. Apr. 01 Tutorial

WEEK 9 **Representability**
Required Reading: BBJ ch. 16
Mon. Apr. 07 Lecture
Tues. Apr. 08 Tutorial

WEEK 10 **Gödel's Incompleteness Thms**
Required Reading: BBJ chs. 17
(BBJ ch. 18 is optional)
Mon. Apr. 14 Lecture
Tues. Apr. 15 Tutorial

WEEK 11 **Second-Order Logic**
Required Reading: BBJ ch. 22
Mon. Apr. 21 Lecture
Tues. Apr. 22 Tutorial

WKS 12–13 **Revision** Mon. Apr. 28 — Fri. May 9

WKS 14–15 **Exams** Mon. May 12 — Fri. May 27